Secondary Flow In Turbomachinery: Theory And Losses

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ABSTRACT

Secondary flows are among the most important source of losses in turbomachinery. How these secondary flow in turbomachines are formed is briefly described in this paper. Estimation of magnitude of secondary vorticities is discussed. There are a number of secondary flow models documenting the progress in our understanding of the secondary flows over the years. Different models describing the secondary flow has been compared. The resulting secondary flow losses are discussed. The turbine efficiency improvement depends on mitigation or reduction of the effect of these losses. The knowledge of these losses provides better idea to reduce these losses. This article provides review on different studies on secondary flow losses in linear turbine cascade. In this paper, tip clearance flows and losses due to that is not covered. Description of secondary flow in cascade is given along with that of vorticity and secondary vorticity, the expression for secondary vorticity is also derived.

NOMENCLATURE

C_D - drag coefficient
L- total lift
C_L- lift coefficient
C – chord length
S – Pitch
ΔP – stagnation pressure loss
ρ – density
Γ- Circulation
Δ- boundary layer thickness
I - Moment of inertia
δ – boundary layer thickness
K.E.-Kinetic energy
W_m- mean flow velocity
α_m-mean flow angle
h- height of the blade
a-speed of sound
ε-turning angle of cascade

1. Introduction

Whenever, the fluid flow occurs in the turbomachines ideally flow field should match with the predicted flow path, which is called as primary path flow but in real flow situations some minor flow is superimposed on that which is called as secondary flow. This additional flow path in turbomachinery is a major source of loss generation. Here, boundary layer developed on the casing and hub walls deflected by the cascade of blades. This fluid flow occurs in axial flow turbines which are used in aircrafts, generation of electricity etc. This is the major contributor of aerodynamic losses and severally affect the efficiency of turbomachinery. The secondary flow in the cascade of turbine blades originate from specifically developing endwall boundary layers and are associated with the presence of longitudinal vortices with a dominant streamwise component of the vorticity. In the literature, the problem with secondary flows, is discussed. There are a number of secondary flow models documenting the progress in our understanding of the secondary flows over the years. Some of these models will briefly be presented in Figure 2. In this paper, the secondary flows in cascades without a tip clearance and relative motion of the blade tips and endwall will be discussed. A few landmark models explaining the development of secondary flows are illustrated in Fig. 2. The main type of secondary flow is the induced recirculating flow, which leads to the formation of a passage vortex.

Secondary flows are an important source of losses in turbomachines, especially in cascades. Due to the complex nature of secondary flows, the evaluation of losses is not an easy task. New formulas and methods are needed to calculate secondary losses. Note that secondary flow phenomena occur in all axial turbomachines. In turbines they can be stronger because of the large amount of flow turning and the high cross-passage pressure gradients that exist. However, in compressors, secondary flow effects are often more apparent and can have greater consequences because of the thick boundary layers on the annulus walls and the highly adverse pressure gradients in the streamwise direction.

Classification of secondary flows-

B. Lakshminarayana (Horlock 1963) classified the secondary flows as one due to rectilinear cascade and other due of tip clearance. In addition, due to relative motion between blade and wall there are some additional losses-

    i.   Cascade secondary flow due to turning of the mainstream.

    ii.   Mainstream secondary flow inside the passage due to vortices trailing from the blade. These vortices may arise from variable wing circulation along the blade or trailing filament vorticity.

    iii.   Leakage flow through the tip clearance and the clearance space around shroud rings, if any. (Only the former is dealt with in this paper)

    iv.   The cross flows in the annulus boundary layer due to rotation and the scraping effect of the blades.

    v.   (a) The inward radial flows due to radial pressure gradients (in the stator).
               (b) The outward radial flows due to rotation of the rotor blades.

2. Description of secondary flow in cascade

When a fluid particle possessing rotation is turned by a cascade, its axis of rotation is deflected in a direction perpendicular to the direction of turning. The rotation of the fluid particles is known as vorticity, which is a vector quantity with a direction along the axis of rotation. The result of deflecting the axis of rotation is a component of vorticity in the direction of the flow streamlines, and whenever this occurs there are secondary flows.(Dixon and Hall 1978)

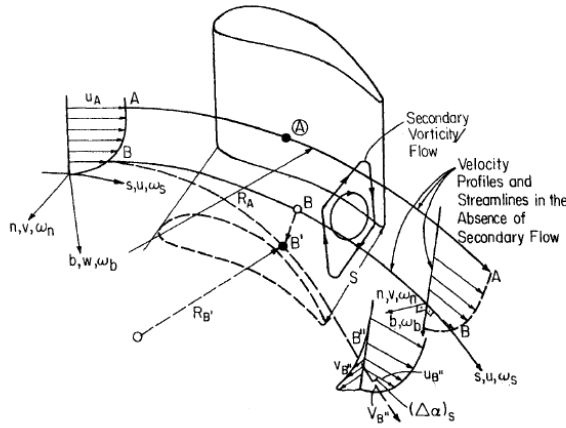Let us consider a 2-D cascade blade row and inlet velocity profile as shown in the figure 1(a).

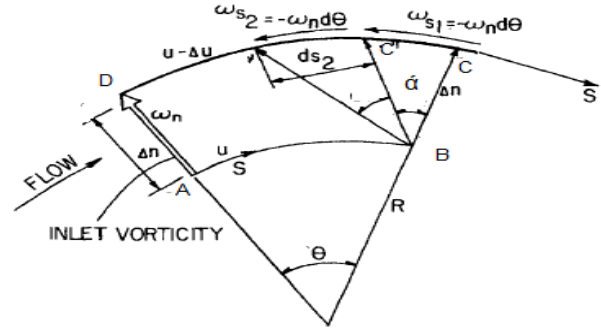**Figure1(a)** Secondary flow phenomenon

Figure 1(b) Estimate of secondary vorticity

Assumption-

Neglect the viscosity effects and velocity variation in the n direction and assume the flow to be incompressible and steady.

AAA represents the streamline where flow is uniform and BBB represents the streamline where flow is in the shear layer. The pressure gradient in the normal direction (n) is balanced by the centripetal acceleration at point A along the streamline AAA.

$$\left(\frac{\partial p}{\partial n}\right)_A = \frac{\rho u_A^2}{R_A} \tag{1}$$

Where R_A is the radius of streamline AAA at point A, where total streamwise velocity is u_A . If the boundary layer approximation is invoked, the pressure gradient normal to the sidewall for the streamline AAA and BBB will be same, thus we obtain.

$$\left(\frac{\partial p}{\partial n}\right)_A = \left(\frac{\partial p}{\partial n}\right)_B = \frac{\rho u_A^2}{R_A} > \frac{\rho u_B^2}{R_B} \tag{2}$$

Because, $u_A > u_B$ and $R_A = R_B$. It is clearly visible that there will be and imbalance between normal pressure gradient and the centripetal acceleration thus the streamline in the shear layer will deflect more, thereby developing cross-flow towards suction side. Fluid particle originating from B will follow the path BB'B" with $R_B < R_A$. The cross flow which is the deviation from the design or main or primary flow is called as secondary flow.

From continuity considerations there will be spanwise velocity w. It can be proven that for turning duct (neglecting streamwise pressure gradient) ,

$$\frac{\partial u}{\partial s} = 0 \tag{3}$$

Hence,

$$\frac{\partial w}{\partial b} = -\frac{\partial v}{\partial n} \tag{4}$$

. This simple explanation provides a clear physical reasoning for the occurrence of the secondary flow in curved bend or a blade passage. The secondary flow gives rise to secondary vortices. These deviation in the primary flow results in

$$\omega_s = -\frac{\partial v}{\partial b} + \frac{\partial w}{\partial n} \tag{5}$$

. This secondary vorticity results in development of secondary flow in the cross-stream plane. (Laxminarayan Budugar, 1996)

Estimation of magnitude of secondary vorticity-

Let us consider flow through the bend or curved duct as shown in fiure

Assumptions- Total pressure is constant, $d\theta$ is small and Bernoulli plane is undistorted. Thus, the basic flow remain two dimentional. Also consider that the flow in the bend is free vortex flow.

Now,

$velocity\ along\ AB = v$

Velocity along $CD = \dfrac{V.r}{r + dr}$

Time Δt for the fluid particle to travel from A to B becomes,

$$\Delta t = \frac{r.\theta}{v}$$

Hence DC'=

$$\Delta t * \frac{V.r}{r + dr} = \frac{r^2.\theta}{r + dr} \tag{5}$$

Now, , Neglecting dr^2

$$CC' \approx 2\theta dr$$

Again,

$\tan\alpha = C'C/BC$

Hence

$\tan\alpha = 2\theta$

"Circulaton is constant for irrotaional flow", $\Gamma_1 = \Gamma_2$

$$\boldsymbol{\omega_s = 2\theta \frac{d\omega}{dy}} \tag{6} ,$$

This is the magnitude of secondary vorticity.

3. Passage and counter vortex

The boundary layer flow is turned more than the primary flow in the cascade channel, leading to a crossflow from the pressure to suction surface in the endwall boundary layer. A compensating return flow must then occur at a certain distance from the endwall, giving rise to the recirculating flow described by e.g. Hawthorne (1951), which can be seen in Fig. 2a. From this recirculating flow, a passage vortex is formed. Downstream in the blade-to-blade passage, due to the pressure-to-suction side pressure difference, the passage vortex locates near the blade suction surface. As a result of the recirculating flow in the neighboring blade-to-blade passages, a vortex layer is formed at the trailing edge, which is quickly rolled-up downstream into a shed trailing edge vortex. Another element of secondary flows is a horse-shoe vortex. The process of formation of the horse-shoe vortex upstream of the leading edge and its downstream transport was explained by Marchal and Sieverding (1977), Hodson and Dominy (1987), Eckerle and Langston (1987), Langston (2006). The models of this process presented in these papers differ from one another in details only.
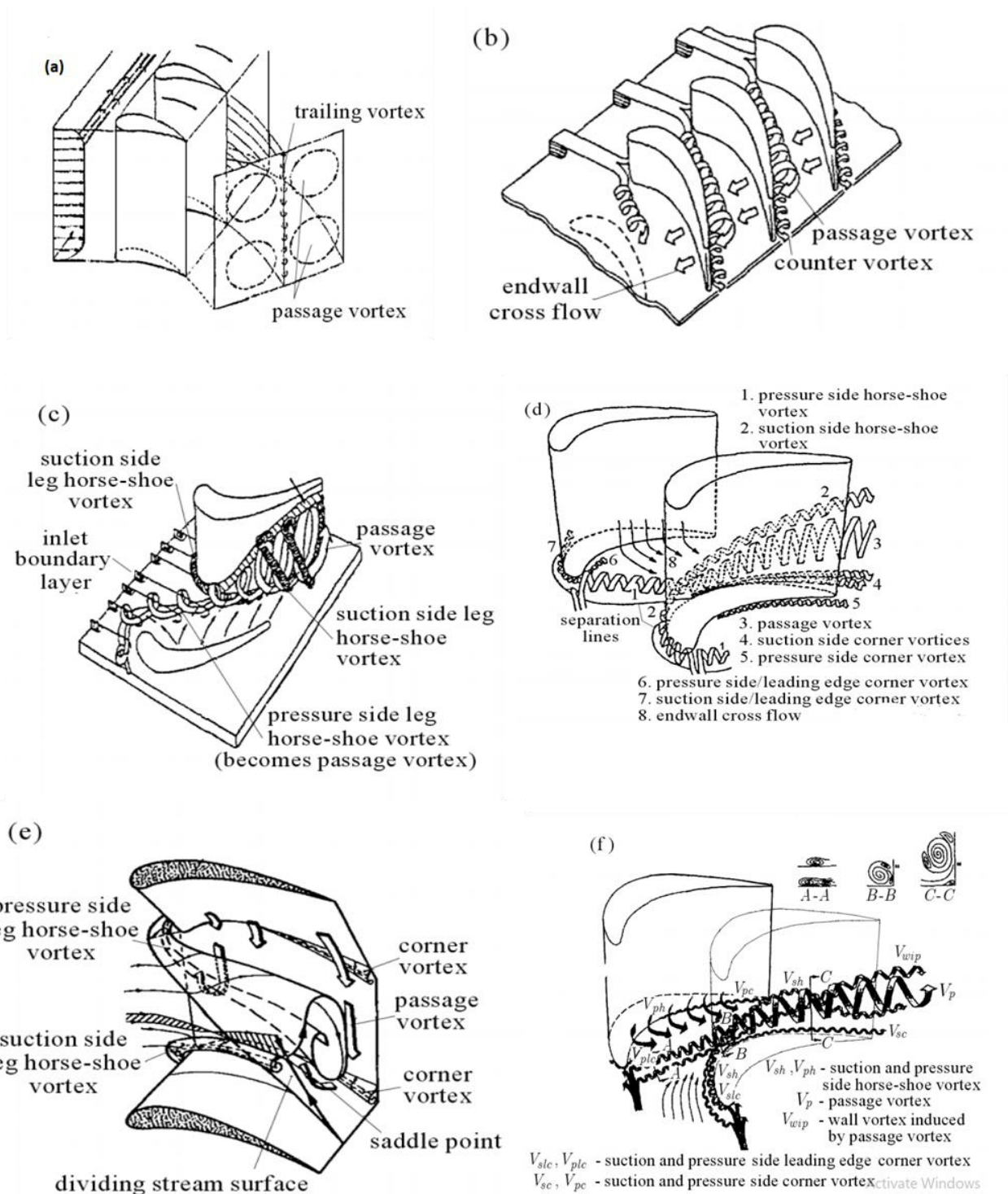
Figure 2 Secondary flow models in turbine cascades: (a) – model of Hawthorne (1955), (b) – model of Langston (1980), (c) – model of Sharma and Butler (1987), (d) – model of Goldstein and Spores (1988), (e) – model of Doerffer and Amecke (1994), (f) – model of Wan

Term Paper ME617A

The boundary layer fluid upstream of the leading edge is decelerated by the adverse pressure gradient and separates at a saddle point s1. The boundary layer fluid elements form a reverse recirculating flow just before the leading edge. This reverse flow separates at another saddle point s2. Sharma O.P. (1986). The upstream boundary layer rolled-up in the recirculating zone flows past the leading edge and is transported downstream in two legs – pressure-side and suction-side leg of the horse-shoe vortex. The suction-side leg of the horseshoe vortex moves near the suction surface of the blade. The pressure-side leg subject to the pressure gradient towards the suction surface moves across the blade-to-blade passage towards this surface. The legs of the horse-shoe vortex move along the lift-off lines that are lines of the saddle points as illustrated in Fig. 2. The location of the horse-shoe vortex lift-off lines, especially that of the pressure-side leg depends on the load of the front part of the blade. For the case of front-loaded profiles with high flow turning in the front part of the blade-to-blade passage, the lift-off line of the horse-shoe vortex pressure-side leg reaches earlier the vicinity of the suction surface than for the case of the aft-loaded profiles.
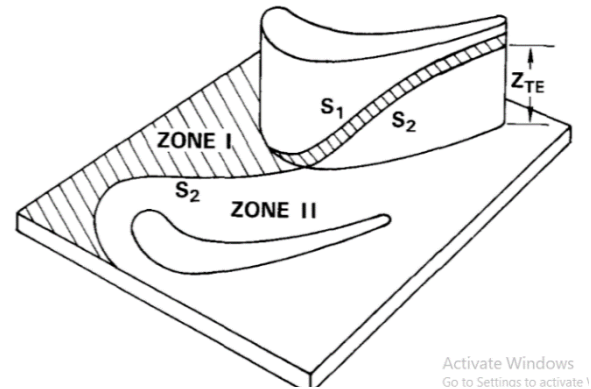

Figure 3 Cascade endwall region separation line by Sharma O.P. et al 1986

Model of Langston (1980) Shown in figure 1(b) explains the transport of horse shoe vortex the pressure side leg of the horse shoe vortex along with the endwall crossflow forms the main recirculating flow but suction side leg stays apart counter rotating with respect to passage vortex. In the model of Sharma and Butler (1987) (Fig. 1c), the suction-side leg of the horse-shoe vortex is wrapped around the passage vortex, whereas in the model of Goldstein and Spores (1988) (Fig. 1d), the suction-side leg locates above the passage vortex and moves together with it. The picture looks similar in the model of Doerffer and Amecke (1994) (Fig. 1e), with a dividing stream surface between the passage vortex and the suction-side leg of the horse-shoe vortex. In point of the suction-side leg of the horse-shoe vortex, Wang et al. (1997) (Fig. 1f) return to the concept of Sharma and Butler (1987) – this leg of the horse-shoe vortex remains wrapped around the passage vortex. In addition to that, both legs of the horse-shoe vortex are formed not from a single vortical structure, but from a pair of alternately dissipating vortices.

4. Estimation of losses

For design of turbomachinery the knowledge about losses is very essential. These losses have been divided into four distinct parts, namely profile losses, secondary flow losses, tip clearance losses and annulus loss.

The profile losses are generated on the airfoil surfaces due to the growth of boundary layers. It is observing that the secondary flow can also affect the profile and annulus losses. The aerodynamic losses so attributed to the endwall- usually termed as secondary flow losses or secondary losses can be as high as 30-50 % of total aerodynamic losses in a blade or stator row(Sharma and Butler 1987). Due to churning of boundary layer near casing annulus loss occurs.

This loss occurs in the regions of flow near the end walls owing to the presence of unwanted circulatory or cross flows. The flow near the end walls give rise to circulatory flows which are mixes with main flow through the blade passage. As a result of this, secondary vortices in the stream wise direction are generated in the blade passages. These vortices try to transport low energy fluid from the pressure side to suction side of the blade passage, thus increasing the possibility of separation of the boundary layer on the suction side.

Secondary flow drag coefficient-

$$\theta = \frac{\Delta W_\theta cos\alpha_m}{W_m} \tag{7}$$

$$L = \rho W_m \Gamma \tag{8}$$

$$L = \rho W_m S(\Delta W_\theta) \tag{9}$$

Hence,

$$C_L = \frac{L}{\frac{1}{2}\rho W_m \, C}$$

$$C_L = 2\frac{S\Delta W_\theta}{CW_m}$$ (10)

$$\frac{\Delta W_\theta}{W_m} = \frac{\theta}{cos\acute{\alpha}_m}$$

$$\omega_s = \frac{CC_L}{S}cos\alpha_m\frac{W_m}{\delta}$$ (11)

Assume the boundary layer to be laminar and the velocity variation in the boundary layer is considered linear.

mass flow rate through boundary layer $= \delta SW_m cos\alpha_m\, \rho$

Kinetic energy of secondary flow $= \frac{1}{2}I\Omega^2$

Where

$I = mr^2$

Let us further assume the fluid is rolling with a circular core,

Hence,

$$\prod r^2 = \delta S$$

$r \approx \sqrt{\delta S}$

Hence, K.E. of the secondary flow becomes,

$K.E.\,\alpha\,\delta SW_m\,cos\alpha_m(\delta S)$

$K.E.\,\alpha\,\rho W_m{}^3 C^2 C_L^2 cos\alpha_m{}^3$ (1

Mass flow through cascade

$m_c = \rho hSW_m\,cos\alpha_m$

Hence,

The secondary flow loss per unit mass flow rate $\alpha\,\dfrac{\rho W_m{}^3 C^2 C_L^2 cos\alpha_m{}^3}{\rho hSW_m\,cos\alpha_m}$

Loss per unit volume $\boldsymbol{\alpha}\,\rho W_m{}^2\dfrac{C^2}{hS}C_L^2 cos\alpha_m{}^2 = \Delta p_0$

$\Delta p_0 =$ (total pressure loss)

For drag coefficient

$$C_{D_s} = \frac{D}{\frac{1}{2}\rho W_m{}^2 C}$$ (13)

$$C_{D_s} = \frac{\Delta p_0 Scos\alpha_m}{\frac{1}{2}\rho W_m{}^2 C}$$

$$C_{D_s} = K\frac{C}{h}C_L^2 cos\alpha_m{}^3$$ (14)

Empirical relation, $C_{D_s} = 0.018 C_L^2$

Cascade secondary losses-

Laxminaryana and Horlock, (1962) has defined that, due to complexities associated with the secondary flows empirical relations are used to account for the three dimensional secondary flow effects. Hawthorne's as expression for the drag coefficient due to the kinetic energy in the secondary flow is –

$$C_{D_s} = \frac{\varepsilon^2 (S/C)^2 cos\alpha_m^2}{\frac{L}{C}} f(\frac{\delta}{S'}) \tag{15}$$

The function of (δ/S') can be obtained from the plots given in Ref 5. Experiments show that for a cascade with no separation the three-dimensional loss is very small and is only a fraction of the kinetic energy in secondary flow.

Vavra's 8 empirical relation for the induced drag coefficient due to secondary flow is

$$C_{D_{Is}} = 0.04 C_L^2 / A \tag{17}$$

Thus we can write the total empirical drag coefficient in a rectilinear cascade as

$$C_D = C_{D_p} + C_{D_{Is}}$$

where C_{D_p} is the drag coefficient in two-dimensional flow, and $C_{D_{Is}}$ is given by equation. Scholz concludes from his experimental results that the secondary-flow losses in a turbine cascade are very small compared with two-dimensional losses, for blades of normal length. For compressor cascades the secondary-flow losses are of the same magnitude as the two-dimensional losses and in some cases larger. It is also concluded that the secondary losses are at maximum for a cascade at S/C = 1. This is in agreement with measurements for a turbine cascade. These experimental investigations are in conflict with analysis which predicts increase in the strength of secondary vortices at increased space chord-ratios.

5. CONCLUSION

- Secondary flow is highly three-dimensional and undesirable flow occurs due to strong interaction of boundary layers ahead of the turbine blade leading edge.
- All main forms of secondary flows meet at the suction surface of the blade
- Passage and counter vortex are defined. So that, by understanding the mechanism of formation, loss can be produced or the effect is mitigated.
- Several classical secondary models of secondary flows in turbine cascades has been presented and compared.
- Losses due to secondary flows are explained and drag coefficient has been derived for estimation of losses.
- Empirical formulae for cascade secondary losses are also given.

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